A Unified Framework for Joint Power Control and Rate Control of Multi-hop Sensor Networks with Energy Replenishment

Abstract—**Renewable energy sources can be attached to sensor nodes to provide energy replenishment for extending the battery life and prolonging the lifetime of sensor networks. However, for networks with replenishment, conservative energy expenditure may lead to missed recharging opportunities due to battery capacity limitations, while aggressive usage of energy may result in reduced coverage or connectivity for certain time periods. Thus, new power allocation schemes need to be designed to balance these seemingly contradictory goals, in order to maximize sensor network performance. In this work, we study the problem of how to jointly control the data queue and battery buffer to maximize the long-term average sensing rate of a wireless sensor network with replenishment with certain QoS constraints for the data and battery queues. We proposed a unified algorithm structure and analyze the performance of the algorithm for all combinations of finite and infinite data and battery buffer sizes. We also provide a distributed version of the algorithm with provable efficiency based on imperfect scheduling.**

I. Introduction

Wireless sensor networks are widely used in monitoring [1], maintenance [2], and environmental sensing [3]. The lack of easy access to a continuous power source and the limited lifetime of batteries have hindered the wide-scale deployment of such networks. However, new and exciting developments in the area of renewable energy [4] [5] provide an alternative to a limited power source, and may help alleviate some of the deployment challenges. These renewable sources of energy could be attached to the nodes and would typically provide energy replenishment at a slow rate (compared to the rate at which energy is consumed by a continuous stream of packet transmissions) that could be variable and dependent on the surroundings.

Energy management in networks equipped with renewable sources is substantially and qualitatively different from energy management in traditional networks. For example, conservative energy expenditure could lead to (i) instability of the data queue because the energy is not being fully used to transmit at high enough data rates, or (ii) missed recharging opportunities because the battery buffer is full. On the other hand, an over aggressive use of energy may lead to lack of coverage or connectivity for certain time periods, which could hurt the application's requirements. Further, if the battery of a node discharges completely, it could be temporarily incapable of transferring time-sensitive data to the sink. This may have undesirable consequences for many applications. Thus, new techniques and protocols must be developed for networks with replenishment to balance these seemingly contradictory goals.

In this paper, we consider single link communication, in which the transmitting node has an infinite data buffer that holds the incoming variable-rate sensing data and a finite battery buffer, which is being replenished at a variable rate. One may experience instability of the data queue or frequent occurrences of battery discharge (batteries becoming empty) if power consumption and sensing rates are not managed properly. We investigate the problem of maximizing the long-term average sensing rate, subject to data queue stability and constraints on the desired rate of visits to zero battery state. We provide a simple joint rate control and power management scheme, and show that the performance of our scheme is close to optimum.

While the problem of energy management in sensor networks has seen considerable attention, there are relatively few works [6], [7], [8], [9], [10], [11] that also include energy replenishment. In [6], the authors consider the problem of dynamic node activation in rechargeable sensor networks. They provide a distributed threshold policy that achieves a performance within a certain factor of the optimal solution for a set of sensors whose coverage area overlap completely. In [7], the authors study the problem of computing lexicographically maximum data collection rate for each node such that no node will be out of energy. In [8], the authors consider the problem of energy-aware routing with distributed energy replenishment. They provide an algorithm that achieves a logarithmic competitive ratio and is asymptotically optimal with respect to the number of nodes in the network. However, the problems considered in these works is very different from the one considered here in that the coupling between the data and battery queue is not taken into account. Prior work on power allocation for wireless networks without replenishment has been widely studied, e.g., [12], [13], [14]. In [12], the authors develop approximated algorithms to minimize average allocated power, or maximize throughout given average power constraint, and at the same time keep the data queue stable. The approximation improves at the cost of increasing the data queue length and queueing delay. In [13], [14] the authors assume that the data buffer is large enough so that packet loss does not occur and provide a dynamic programming based solution. Most of these works assume a constant energy supply, i.e., there is no battery issue. In this paper, we explicitly model energy replenishment and consider jointly managing the data and battery buffers. This coupling between the data and battery buffers is what makes the problem notoriously difficult

to solve using standard optimization based approaches. For example, unlike prior works that utilize the fact that a static allocation policy is optimal, and then develop dynamic policies based on their performance against a dynamic policy, it isn't even clear whether a static policy would be optimal in our setting. Specifically, with a battery buffer, there is an additional energy constraint that the allocated energy should be within the battery state, and this constraint is even more difficult than the average power constraint.

The main contributions of the paper are as follows:

- We formulate a sensing rate maximization problem with QoS constraints on both data and battery queues. Due to the coupling between the battery and data buffers it is unlikely that a stationary policy optimal, hence traditional resource allocation techniques do not directly apply. Further, dynamic programming based solutions result in prohibitively high complexity, even for the single-link scenario. Nonetheless, we are able to develop a simple and unified framework for all combinations of finite and infinite data and battery buffer sizes, and our algorithm is provably efficient.
- We extend the algorithm from a single-link case to a multi-hop network under node-exclusive interference model with practical settings and develop an efficient distributed algorithm based on imperfect scheduling.

II. Single Link Model

We first consider a single link control model in this paper, as illustrated in Figure 1. We assume a time slotted system and in time slot t, the amount of data available for the sensor node to sense is denoted by $A(t)$, which is upper bounded by $0 < A_{\text{max}} < \infty$. In the same time slot, the actual amount of data the node senses and places in the data buffer is $R(t)$. The amount of energy the node spends at time t for data transmission is $P(t)$, which is upper bounded by $0 < P_{\text{peak}} < \infty$ and the achievable data rate at that power level is $\mu(P(t))$, where we assume $\mu(\cdot)$ to be monotonically increasing, reversible and differentiable on the half real line $\Re^+ \bigcup\{0\}$. The node has a battery of size B_b (either $B_b < \infty$ or $B_b = \infty$) with zero initial battery state, where the energy for transmission is coming from and the energy in the battery is being replenished at a variable instantaneous rate $r(t)$. The energy state of the battery and the data state of the data buffer at time t is $q_b(t)$ and $q_d(t)$, respectively. The data buffer has size B_d (either $B_d < \infty$ or $B_d = \infty$). Under this setting, we describe the objective in the following section.

A. Problem Formulation

Our general objective is to maximize the long-term average sensing rate, subject to the QoS constraints on both data and battery queues:

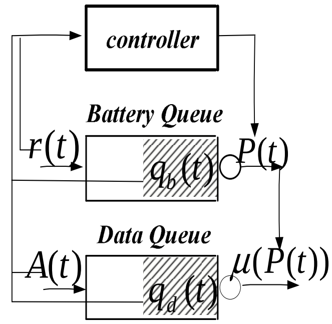


Fig. 1. Single Link Control Model

$$(A) \qquad \max_{P,R} \liminf_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} R(t)$$

$$\text{s.t. } q_d(t+1) = \min\left[(q_d(t)-\mu(P(t)))^+ + R(t), B_d\right], \quad (a)$$

$$q_b(t+1) = \min\left[q_b(t) - P(t) + r(t), B_b\right], \quad (b)$$

$$0 \le R(t) \le A(t), \quad (c)$$

$$0 \le P(t) \le \min[q_b(t), P_{\text{peak}}], \quad (d)$$

$$\limsup_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} q_d(t) < \infty \ (B_d=\infty), \text{ or}$$

$$p_d \le \eta_d \ (B_d < \infty), \quad (e)$$

$$p_o \le \eta_o, \quad (f)$$

where $(\cdot)^+ = \max[\cdot, 0]$, $R = \{R(0), R(1), \ldots, R(T-1), \ldots\}$ is the actual sensing data vector, $P = \{P(0), P(1), \ldots, P(T-1), \ldots\}$ is the allocated power vector, and

$$p_d = \limsup_{T\to\infty} \frac{\sum_{t=0}^{T-1} D(t)}{\sum_{t=0}^{T-1} R(t)}, \quad (1)$$

$$p_o = \limsup_{T\to\infty} \frac{\sum_{t=0}^{T-1} I_o(t)}{T} \quad (2)$$

are the long-term data loss ratio with an upper bound η_d, and the frequency of visits to zero battery state with given threshold η_o respectively, where

$$D(t) = ((q_d(t) - \mu(P(t)))^+ + R(t) - B_d)^+, \quad (3)$$

$$I_o(t) = \mathbf{1}_{\textit{battery state '0' is visited from higher states in slot t}} = \begin{cases} 0 & \text{if } P(t)=0 \text{ or } P(t) < q_b(t) \\ 1 & \text{otherwise} \end{cases} \quad (4)$$

are the amount of data loss in slot and the indicator that the battery discharges completely in time slot t, respectively. Note that, if the battery is completely discharged at time t and there is no replenishment at slot t, we do not consider a complete discharge event occurring at time $t+1$ as well. We do not assume ergodicity of the system parameters, but if they are ergodic, then p_o represents the actual probability of a complete discharge event as $t \to \infty$.

In Problem (A), constraints (a) and (b) describe how the data and battery queues evolve, respectively. Especially, if $B_d = \infty$, (a) can be simplified to

$$q_d(t+1) = (q_d(t) - \mu(P(t)))^+ + R(t), \quad (a')$$

and if $B_b = \infty$, (b) can be simplified to

$$q_b(t+1) = q_b(t) - P(t) + r(t). \qquad (b')$$

Constraint (c) bounds the actual amount of sensed data $R(t)$ by the available amount of data $A(t)$ in slot t. Constraint (d) states that we cannot oversubscribe the energy that is unavailable in the battery nor can we exceed the peak power level. Constraint (e) is the QoS constraint for data queue: if $B_d = \infty$, we need to keep the data queue stable, and if $B_d < \infty$, the data loss ratio is required under a given threshold. Constraint (f) is the battery QoS constraint of the desired battery discharge rate η_o.

Problem (A) has an inventory control structure, and typically such a structure can be solved optimally using dynamic programming, albeit with high complexity. Furthermore, depending on the exogenous processes $\{A(t),\ t \geq 0\}$ and $\{r(t),\ t \geq 0\}$, Problem (A) may not have a feasible solution, i.e., there exists no power allocation policy $\{P(t),\ t \geq 0\}$ that satisfies all constraints simultaneously. In this paper, our purpose is to develop a simple algorithm, which performs arbitrarily close to the performance of the optimal power allocation, whenever a feasible solution exists. To achieve that purpose,

- we define *virtual queues* for both the data and battery buffer to avoid the difficulties involved in dealing with the data loss and battery discharge probability directly. We show that keeping the virtual queues stable ensures that the constraints on p_d and p_o are met.
- we design an power allocation scheme based on simple index policies and show that our scheme keeps both virtual queues stable and at the same time performs arbitrarily close to the optimal performance.
- we generalize our algorithm to the multihop scenario and develop distributed algorithms.

B. Virtual Queues

We define $\tilde{q}_d$ and $\tilde{q}_b$ as the virtual data and battery queues. The virtual queues evolve according to the following Lindley's queue evolution equations:

$$\tilde{q}_d(t+1) = (\tilde{q}_d(t) - \eta_d R(t))^+ - \mu(P(t)) + R(t) + I(t), \qquad (5)$$

$$\tilde{q}_b(t+1) = (\tilde{q}_b(t) - \eta_o)^+ + P(t) - r(t) + M(t) + I_o(t), \qquad (6)$$

where $I(t) = (\mu(P(t)) - q_d(t))^+$ is the amount of sent idle packets when there is no enough data to transmit using the allocated energy, $M(t) = (q_b(t) - P(t) + r(t) - B_b)^+$ is the amount of missed replenishing energy due to full battery when $B_b < \infty$, and $I_o(t)$ is defined in Equation (4). Note that if $B_b < \infty$, $M(t) = 0$ and Equation (6) reduces to

$$\tilde{q}_b(t+1) = (\tilde{q}_b(t) - \eta_o)^+ + P(t) - r(t) + I_o(t). \qquad (7)$$

Without loss of generality, the initial state $\tilde{q}_b(0)$ and $\tilde{q}_b(0)$ can be set to be zero. The following proposition shows that if the virtual queues $\tilde{q}_d(t)$, $\tilde{q}_b(t)$ and the actual battery queue $q_b(t)$ are all strongly stable, p_d and p_o are guaranteed to meet their constraints.

Proposition 1: If the virtual queues $\tilde{q}_d(t)$, $\tilde{q}_b(t)$ and the actual battery queue $q_b(t)$ are both strongly stable, i.e.,

$$\limsup_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} \tilde{q}_d(t) < \infty,$$

$$\limsup_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} \tilde{q}_b(t) < \infty,$$

$$\limsup_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} q_b(t) < \infty,$$

then $p_d \leq \eta_d$ and $p_o \leq \eta_o$.

The proof of this result can be found in Appendix A. Next, we present our scheme.

III. Joint Power Allocation and Admission Control Algorithm

A. Algorithm

The algorithm consists of two components: a *rate control* component and a *power allocation* component. Both components are *index policies*, i.e., the solutions are memoryless and they depend on the instantaneous values of the system variables.

Rate Control (RC):

We define $0 < V < \infty$ to be the control parameter of our algorithm. Let $Q_d(t) = q_d(t)$ when $B_d = \infty$, and let $Q_d(t) = \tilde{q}_d(t)$ when $B_d < \infty$. If $Q_d(t) \leq \frac{V}{2}$, the transmitting node chooses to sense all the available data packets, i.e., $R(t) = A(t)$. Otherwise, $R(t) = 0$.

Power Allocation (PA):

Solve the following optimization problem

$$\max_{P(t)\in\Pi(t)} Q_d(t)\mu(P(t)) - \tilde{q}_b(t)P(t), \qquad (8)$$

where $\Pi(t) = \{P(t) : 0 \leq P(t) \leq \min[q_b(t), P_{\text{peak}}]\}$ is a compact and nonempty set. Allocate $P(t)$.

The set, $\Pi(t)$ of possible power allocations guarantees Constraint (d) on $P(t)$ in Problem (A). If $\mu(\cdot)$ is concave, the objective function is a concave function of $P(t)$. Consequently, *PA* solves a simple convex optimization problem in each time slot. The positive term $Q_d(t)\mu(P(t))$ can be viewed as a utility of power allocation $P(t)$ and the term $\tilde{q}_b(t)P(t)$ can be viewed as its associated cost. When the control parameter V is chosen to be large, $Q_d(t)$ tends to be large according to *RC*, and *PA* tries to allocate higher power $P(t)$ to increase the utility, whereas, when the virtual battery queue length $\tilde{q}_b(t)$ is large, *PA* avoids allocating a high amount of power to reduce the cost. Thus, this index policy of *PA* can be viewed as a *greedy profit maximization* scheme.

B. Performance Analysis

Recall that $A(t)$ is the available amount of sensing data and $R(t)$ is the actual amount of data the transmitting node chooses to sense. Clearly, using the rate controller, we make sure that the data queue remains within a certain bound and this has a positive effect on the battery as well, since certain proportion

of the data packets are not allowed into the transmitting node. The natural question one would ask here is, whether our rate controller rejects too many packets in the first place to *synthetically* meet the constraints. In the following theorem, we show that this is not the case. Indeed, if there exists a solution, $\lambda^* = \liminf_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} R^*(t)$ to Problem (A) for the arrival process $\{A(t),\ t \geq 0\}$ and the replenishment process $\{r(t),\ t\geq 0\}$, then the sensing rate associated with ***RC*** and ***PA*** can be made asymptotically closer to λ^* by increasing the control parameter V with increasing B_d and B_b. We use the notation $y = O(x)$ to represent y going to 0 as x goes to 0.

Theorem 1: If
(1) $\mu(\cdot)$ is concave on $\Re^+ \bigcup\{0\}$, and its slope at 0 satisfies[1] $0 \leq \beta = \mu'(0) < \infty$,
(2) $r(t) > 0,\ \forall t \geq 0$, and $\bar{r} = \limsup_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} r(t) \leq P_{\text{peak}}$,
(3) A feasible solution to Problem (A) exists and the optimal instantaneous sensing rate is $R^*(t)$,
then the joint power allocation and admission control algorithm (with ***RC*** and ***PA***) achieves:

$$Q_d(t) \leq \frac{V}{2} + A_{\max}, \quad \forall\ t \geq 0 \tag{9}$$

$$\tilde{q}_b(t) \leq \beta(\frac{V}{2} + A_{\max}), \quad \forall\ t \geq 0 \tag{10}$$

$$q_b(t) < \infty, \quad \forall\ t \geq 0 \tag{11}$$

$$\liminf_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} R(t) \geq \liminf_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} R^*(t) - O(\frac{1}{V}) - \eta_o(\mu_{\max} + \beta) - g(V, B_d, B_b), \tag{12}$$

where $\mu_{\max} = \mu(P_{\text{peak}})$ is the upper bound for the transmission rate, and

$$g(V, B_d, B_b) = \begin{cases} 0, & \text{if } B_d < \infty,\ B_b < \infty \\ O(\frac{(\frac{\beta}{2}V - B_b)^+}{V}), & \text{if } B_d = \infty,\ B_b < \infty \\ O(\frac{(\frac{V}{2} - B_d)^+}{V}), & \text{if } B_d < \infty,\ B_b = \infty \\ O(\frac{(\frac{\beta}{2}V - B_b)^+}{V}) + O(\frac{(\frac{V}{2} - B_d)^+}{V}), & \\ & \text{if } B_d < \infty,\ B_b < \infty. \end{cases}$$

The proof of Theorem 1 can be found in Appendix B. From Equation (9), Equation (10) and Equation (11), the virtual queues $\tilde{q}_d(t),\ \tilde{q}_b(t)$ and the actual battery queue $q_b(t)$ are all strongly stable. Thus, by Proposition 1, $p_d \leq \eta_d$ and $p_o \leq \eta_o$. In Theorem 1, V is a finite tunable approximation parameter that controls the efficiency of the algorithm. Observe Equation (12), the term $\eta_o(\mu_{\max} + \beta)$ captures the influence of battery outage, and it is small since the battery outage threshold η_o is usually set to be very small to avoid network disconnection. $g(V, B_d, B_b)$ represents the asymptotical property of the gap, i.e., if at least one of the buffer size is finite, as V increases, the buffer size should increase accordingly to keep the gap small.

[1]For instance, consider the additive white Gaussian noise channel capacity, $\mu(P) = \log(1 + P/N_0)$, where N_0 is the two sided noise power spectral density.

IV. Joint Energy and Queue Management in Multihop Networks

A. Formulation

We consider a multihop wireless sensor network with N nodes and L links. Each node $n \in \mathcal{N} = \{1, 2, \ldots, N\}$ is attached to power sources for replenishment. Let $A_n^e(t)$ and $R_n^e(t)$ denote the amount of available data for sensing and the actual amount sensing data, to node n that are destined to node e in slot t. Assume that each node n maintains a infinity data buffer with state $q_d^{n,e}(t)$ for flows destined to e, and also maintains a finite battery buffer with size B_b^n and state $q_b^n(t)$. Let $r_n(t)$ denote the replenishment at node n in time slot t. The transmit power is chosen to be $P_l(t)$ over link l. In the formulation, we assume that the power the receiving node consumes to receive and decode the packet is identical to $P_l(t)$ as well. The sole reason for this is simplicity and the generalization to the asymmetric case is straightforward. We use the node-exclusive interference model. Under this model, a node can only receive from or transmit to at most one node at any time slot, and in each time slot t, with the assigned power $P_l(t)$, the achieved data rate at link l is $\mu_l(P_l(t))$ in that time slot, where the rate function $\mu_l(\cdot)$ is a non-decreasing and differentiable function satisfying $\mu_l(0) = 0$. Let p_o^n be the frequency of visits to the zero battery state for node n. Let Ω_n and Θ_n denote the set of directed links originated from node n and terminate at node n, respectively. In a multihop network under node-exclusive model, we formulate the joint queue and energy management problem as follows:

$$(B) \qquad \max_{\vec{P},\vec{R}} \liminf_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1}\sum_{n,e\in\mathcal{N}} R_n^e(t)$$

s.t. $\vec{P}(t)$ satisfies the node-exclusive model, $\quad (a)$

$$q_d^{n,e}(t+1) = (q_d^{n,e}(t) - \sum_{l\in\Omega_n}\mu_l^e(P_l(t)))^+ + R_n^e(t) + \sum_{l\in\Theta_n}\mu_l^e(P_l(t)), \quad n \neq e, \tag{b}$$

$$q_b^n(t+1) = \min\left[q_b^n(t) - \sum_{l\in\Omega_n\cup\Theta_n} P_l(t) + r_n(t), B_b^n\right], \tag{c}$$

$$0 \leq \sum_{l\in\Omega_n\cup\Theta_n} P_l(t) \leq q_b^n(t),\ 0 \leq P_l(t) \leq P_{\text{peak}}, \tag{d}$$

$$\sum_{e=1}^{N}\mu_l^e(P_l(t)) = \mu_l(P_l(t)),\ R_n^e(t) \leq A_n^e(t), \tag{e}$$

$$\limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1} q_d^{n,e}(t) < \infty, \quad n \neq e, \tag{f}$$

$$p_o^n \leq \eta_o^n, \tag{g}$$

where η_o^n is the desired upper bounds for p_o^n, $\vec{P}(t)$ is the power assignment vector for all links in slot t, $\vec{P}$ is the power assignment for all links over all time slots, and $\vec{R}$ is the actual sensing data vector for all node-destination pairs over all time slots. In Problem (B), the objective is to maximize the long-

term average total sensing rate to all nodes destined to all destinations.

In Problem (B), (a) is the interference constraint. constraints (b) and (c) describe how the data and battery queues evolve, respectively. Note that the destination node of each flow does not need to maintain a data buffer for that flow, as indicated in (b). Constraints (d) are the energy conservation equations stating that we cannot oversubscribe the energy that is unavailable in the battery nor can we exceed the peak power level. Constraints (e) are the rate conservation equations that bound the actual amount of sensed data $R(t)$ by the available amount of data $A(t)$, and share the transmission rate of a link among all the destinations in slot t. Constraint (f) is the QoS constraint for data queue: we need to keep all the data queues stable. Constraint (g) is the battery QoS constraint of the desired battery discharge rate η_o.

Similarly, we define virtual queues for all $n \in \mathcal{N}$

$$\tilde{q}_b^n(t+1) = (\tilde{q}_b^n(t) - \eta_o^n)^+ + \sum_{l \in \Omega_n \cup \Theta_n} P_l(t) - r_n(t) + M_n(t) + I_o^n(t), \quad (13)$$

where $M_n(t) = (q_b^n(t) - \sum_{l \in \Omega_n \cup \Theta_n} P_l(t) + r_n(t) - B_b^n)^+$ is the amount of missed replenishment and

$$I_o^n(t) = \textit{indicator that battery state '0' is visited from higher states in slot } t \textit{ for node } n$$

$$= \begin{cases} 0 & \text{if } \sum_{l \in \Omega_n \bigcup \Theta_n} P(t) = 0 \text{ or } \sum_{l \in \Omega_n \bigcup \Theta_n} P_l(t) < q_b^n(t) \\ 1 & \text{otherwise} \end{cases}$$

Next, we generalize the main results and algorithms *RC* and *PA* to the multihop scenario.

Corollary 1: If all the virtual battery queues $\tilde{q}_b^n(t),\ \forall n \in \mathcal{N}$ are strongly stable, we have $p_o^n \leq \eta_o^n,\ \forall n \in \mathcal{N}$.
The proof is identical to the single hop scenario and can be found in Appendix C. We give the algorithm in the following section.

B. Algorithm

Multihop Rate Control (MRC):
Maximum Weighted Matching Algorithm: If $q_d^{n,e}(t) \leq \frac{V}{2}$, node n chooses to sense all the available data packets, i.e., $R_n^e(t) = A_n^e(t)$; otherwise, reject all the arrivals, i.e., $R_n^e(t) = 0$.
Maximal Matching Algorithm: If $q_d^{n,e}(t) \leq V$, node n chooses to sense all the available data packets, i.e., $R_n^e(t) = A_n^e(t)$; otherwise, reject all the arrivals, i.e., $R_n^e(t) = 0$.

Multihop Power Allocation (MPA):
First define

$$\gamma_l^e = \begin{cases} \gamma & \text{if } rec(l) \neq e \\ 0 & \text{otherwise} \end{cases}$$

where $\gamma > 0$ is some constant for the sake of the algorithm. Let $e_l(t) = \arg\max_e \{q_d^{tran(l),e} - q_d^{rec(l),e} - \gamma_l^e\}$ be the flow on that link l puts all the rate, and $w_l(t) = \max[q_d^{tran(l),e_l(t)}(t) - q_d^{rec(l),e_l(t)}(t) - \gamma_l^{e_l(t)}, 0]$ is related to the modified differential backlog. This means that no node can transfer a flow to a relay node that is not the destination of the flow unless the differential backlog of the data queues between these two nodes is no less than a fixed value γ. Here, $tran(l)$ and $rec(l)$ denote the transmitting and receiving node of link l, respectively.

For each link l, solve

$$\max_{P_l(t) \in \Pi_l(t)} w_l(t)\mu_l(P_l(t)) - (\tilde{q}_b^{tran(l)}(t) + \tilde{q}_b^{rec(l)}(t))P_l(t) \quad (14)$$

where $\Pi_l(t) = \{P_l(t) : 0 \leq P_l(t) \leq \min\left[q_b^{tran(l)}(t), q_b^{rec(l)}(t), P_{\text{peak}}\right]\}$. Let $P_l(t)$ be the solution. With the calculated power $P_l(t)$, let $W_l(t) = w_l(t)\mu_l(P_l(t)) - (\tilde{q}_b^{tran(l)}(t) + \tilde{q}_b^{rec(l)}(t))P_l(t)$ be the weight on link l.

For the whole network,
Maximum Weighted Matching Algorithm: link l has weight $W_l(t)$, then the weight of a matching $\mathcal{M}$ is $W_{\mathcal{M}}(t) = \sum_{l \in \mathcal{M}} W_l(t)$. The network chooses a maximum weighted matching in a centralized manner, the links in the chosen matching become active with the calculated transmitting power, and other links are not activated.

Maximal Matching Algorithm: the network chooses a maximal matching in a fully distributed manner as in [15], the links in the chosen matching become active with the calculated transmitting power, and other links are not activated.

Routing:
When $w_l(t) > 0$, transmit for flow that is destined to $e_l(t)$ with rate $\mu_l(P_l(t))$, i.e., $\mu_l^{e_l(t)}(P_l(t)) = \mu_l(P_l(t))$ and $\mu_l^e(P_l(t)) = 0,\ \forall e \neq e_l(t)$.

MAC and routing can be done by each node independently. Note that *MPA* has two different algorithms, the maximum weighted matching based algorithm is centralized and the maximal matching based algorithm is fully distributed. We give our main theorem for the multihop scenario:

Theorem 2: If
(1) $\mu_l(\cdot)$ is concave on $\Re^+ \bigcup \{0\}$, and its slope at 0 satisfies $0 \leq \beta = \mu_l'(0) < \infty,\ \forall l \in \mathcal{L}$,

(2) $\forall n \in \mathcal{N}$: $r_n(t) > 0,\ \forall t \geq 0$, and $\bar{r}_n = \limsup_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} r_n(t) \leq P_{\text{peak}}$,

(3) A feasible solution to Problem (B) exists and the optimal instantaneous sensing rate vector is $\vec{R}^*(t)$,

then the maximum weighted matching based joint rate control *MRC*, power allocation *MPA*, and routing algorithm

achieves:

$$q_d^{n,e}(t) \le \frac{V}{2} + A_{\max}, \tag{15}$$

$$\tilde{q}_b^n(t) \le \beta(\frac{V}{2} + A_{\max}), \tag{16}$$

$$\sum_n \left\{ \liminf_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \sum_e R_n^e(t) \right\} \ge \sum_n \left\{ \liminf_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \sum_e R_n^{e*}(t) - \eta_o^n(\mu_{\max} + \beta) + O(\frac{(\frac{\beta}{2}V - B_b^n)^+}{V}) \right\} - O(\frac{1}{V}). \tag{17}$$

Further, if the variable $r(t),\ \forall t \ge 0$ are independent from slot to slot, and the variable $A(t),\ \forall t \ge 0$ are also independent from slot to slot, then the maximal matching based joint rate control *MRC*, power allocation *MPA*, and routing algorithm achieves:

$$q_d^{n,e}(t) \le V + A_{\max}, \tag{18}$$

$$\tilde{q}_b^n(t) \le \beta(V + A_{\max}), \tag{19}$$

$$\sum_n \left\{ \liminf_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \sum_e R_n^e(t) \right\} \ge \sum_n \left\{ \liminf_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \sum_e \frac{R_n^{e*}(t)}{2} - \frac{\eta_o^n(\mu_{\max} + \beta)}{2} + O(\frac{(\frac{\beta}{2}V - B_b^n)^+}{V}) \right\} - O(\frac{1}{V}). \tag{20}$$

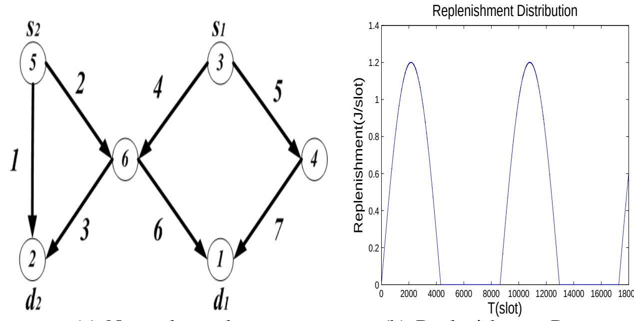
The proof of Theorem 2 can be found in Appendix D. The results can be interpreted similarly as Theorem 1.

V. Numerical Example

In the simulation, we consider a network topology as shown in Figure 2 (a). There are 6 nodes, 7 links, and 2 flows with source-destination pair $(3, 1)$ and $(5, 2)$, respectively. The number of time slots is $T = 10^6$, and the duration of each slot is 10 secs. We use the rate power function $\mu_l(P_l) = 10\log_2(1 + \frac{g_l P_l}{N_l})$ packets/slot $\forall l \in \mathcal{L}$. Let the power of the background noise $N_l = 1.6 \times 10^{-14}W,\ \forall l \in \mathcal{L}$, and the channel gains $g_l = 1.6 \times 10^{-13},\ \forall l \in \mathcal{L}$. Each node is equipped with an infinite data buffer for each flow through it, and a battery buffer of size $B_b = 800J$. The numbers of arrivals $A_n^e(t),\ t \ge 0$, for all nodes and flows, are independent Poisson random variables with mean 20 packets/slot and $A_{\max} = 31$ packets/slot. The replenishment process is periodic with independent Gaussian noise, as shown in Figure 2 (b) (The cycles imitate the daily solar cycles for a solar battery). We also set η_o^n, the threshold of battery outage probability to 0.03 for all $n \in \mathcal{N}$.

We simulate the proposed algorithm for different values of the control coefficient V and compare the results with the optimal value[2]. From Figure 3 (a), we see that as V increases,

[2]The optimal value can be obtained by using dynamic programming. The details are omitted here.



(a) Network topology (b) Replenishment Process

Fig. 2. Simulation Setting

the average total sensing rate keeps increasing and gets closer to the optimal value, which is consistent with Equation (17). From Figure 3 (b), we see that as V increases, the average data queue length (we here only plot the data queue length of node 2 for flow 1) keeps increasing but is upper bounded by the bound we get in Equation (15). From Figure 3 (c), the battery discharge probability (we only plot for node 1) increases to the threshold as V increases. Thus, increasing V can increase the average total sensing rate, but the cost is increasing the average data queue length and battery discharge probability of at least one node.

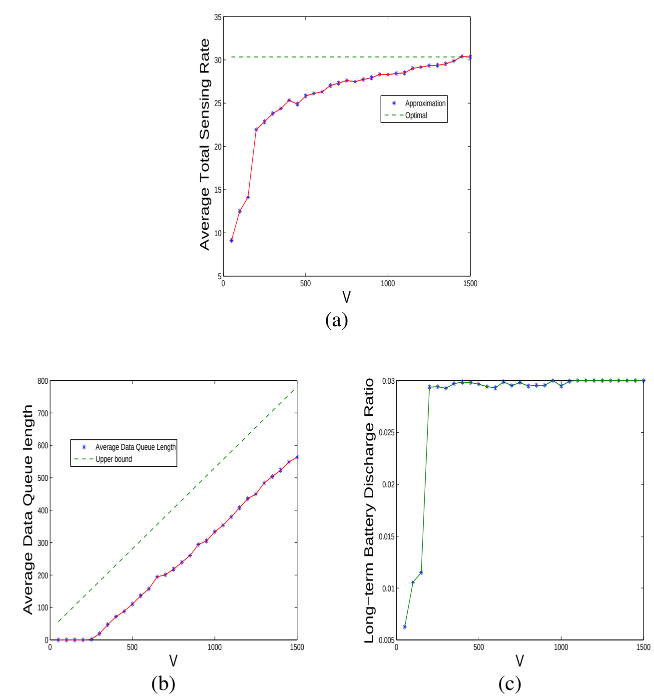
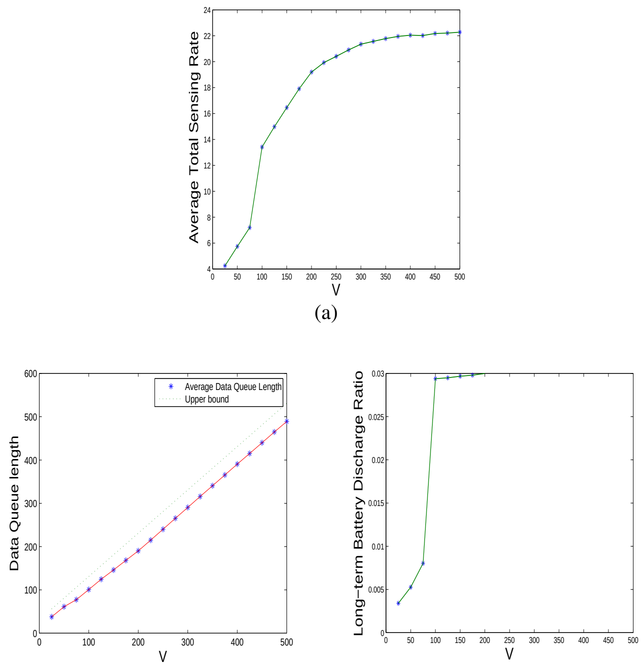


Fig. 3. Performance of the MWM based algorithm. Impact of the control parameter V on (a) the average total sensing rate, (b) average data queue length, and (c) the battery discharge probability

Similarly, from Figure 4 (a), we see that as V increases, the average total sensing rate keeps increasing and gets closer to some value that is larger than half optimum, which is consistent with Equation (20). From Figure 4 (b), we see that as

V increases, the average data queue length (we here only plot the data queue length of node 3 for flow 1) keeps increasing but is upper bounded by the bound we get in Equation (18). From Figure 4 (c), the battery discharge probability (we only plot for node 6) increases to the threshold as V increases.



(a)

(b) (c)

Fig. 4. Performance of the MM based algorithm. Impact of the control parameter V on (a) the average total sensing rate, (b) average data queue length, and (c) the battery discharge probability

VI. Conclusion

In this paper, we studied the problem of energy management in rechargeable wireless sensor networks. Our objective was to maximize the average data sensing rate subject to QoS constraints on both data and battery queues. We provided a simple and unified framework of joint rate control and power allocation for all combinations of finite and infinite data and battery buffer sizes. We showed through both analysis and simulation that the performance of our strategy is close to that of the optimal solution. We extended our algorithm to the multihop scenario and showed that simple extensions of our index schemes for single hop generalize to the multihop scenario with a similar, close-to-optimal performance. We developed a distributed joint rate control, power allocation and routing algorithm for multihop networks under node-exclusive interference model based on imperfect scheduling.

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Appendix

A. Proof of Proposition 1

Using the idea similar to [12], we have the fact that if any queue represented with $Q(t)$ is strongly stable, then $\limsup_{T\to\infty}\frac{Q(T)}{T}\leq 0$. Hence, if $\tilde{q}_d(t)$, $\tilde{q}_b(t)$ and $q_b(t)$ are strongly stable, then $\limsup_{T\to\infty}\frac{\tilde{q}_d(T)}{T}\leq 0$, $\limsup_{T\to\infty}\frac{\tilde{q}_b(T)}{T}\leq 0$ and $\limsup_{T\to\infty}\frac{q_b(T)}{T}=0$.

From Equation (5), we have $\tilde{q}_d(t+1)\geq\tilde{q}_d(t)-\eta_d R(t)+D(t)-D(t)-\mu(P(t))+R(t)+I(t)$. Note that $q_d(t+1)=q_d(t)-\mu(P(t))+I(t)+R(t)-D(t)$. By adding from 0 to $T-1$, dividing by T and taking $\limsup_{T\to\infty}$ on both sides, we have

$$\limsup_{T\to\infty}\frac{\tilde{q}_d(T)}{T}\geq\lim_{T\to\infty}\frac{\tilde{q}_d(0)}{T}-\eta_d\limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}R(t)+\limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}D(t)+\lim_{T\to\infty}\frac{q_d(0)-q_d(T)}{T}.$$

Since $\limsup_{T\to\infty}\frac{\tilde{q}_d(T)}{T}\leq 0$, $\lim_{T\to\infty}\frac{\tilde{q}_d(0)}{T}=0$, $\lim_{T\to\infty}\frac{q_d(0)}{T}=0$ and $\lim_{T\to\infty}\frac{q_d(T)}{T}=0$, so we get

$p_o = \limsup_{T\to\infty} \frac{\sum_{t=0}^{T-1} D(t)}{\sum_{t=0}^{T-1} R(t)} \leq \eta_d$.

Similarly, from Equation (6), we have $\tilde{q}_b(t+1) \geq \tilde{q}_b(t) - \eta_o + P(t) - r(t) + M(t) + I_o(t)$. Note that $q_b(t+1) = q_b(t) - P(t) + r(t) - M(t)$. By adding from 0 to $T-1$, dividing by T and taking $\limsup_{T\to\infty}$ on both sides, we have

$$\limsup_{T\to\infty} \frac{\tilde{q}_b(T)}{T} \geq \lim_{T\to\infty} \frac{\tilde{q}_b(0)}{T} - \eta_o + \limsup_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} I_o(t) + \lim_{T\to\infty} \frac{q_b(0) - q_b(T)}{T}.$$

Since $\limsup_{T\to\infty} \frac{\tilde{q}_b(T)}{T} \leq 0$, $\lim_{T\to\infty} \frac{\tilde{q}_b(0)}{T} = 0$, $\lim_{T\to\infty} \frac{q_b(0)}{T} = 0$ and $\lim_{T\to\infty} \frac{q_b(T)}{T} = 0$, so we get $p_o = \limsup_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} I_o(t) \leq \eta_o$. ∎

B. Proof of Theorem 1

Proof of Equation (9): Note that $I(t) \leq \mu(P(t))$ and $R(t) \leq A_{\text{max}}$. The rate allocation unit *RC* is chosen to satisfy Equation (9).

Proof of Equation (10): Since $\mu(\cdot)$ is concave on $\Re^+ \bigcup\{0\}$, we have $\mu(P(t)) \leq \mu(0) + \beta P(t)$ for $P(t) \in \Pi(t),\ \forall t \geq 0$, where $0 \leq \beta = \mu'(0) < \infty$. Then, $Q_d(t)\mu(P(t)) - \tilde{q}_b(t)P(t) \leq Q_d(t)\mu(0) + \beta Q_d(t)P(t) - \tilde{q}_b(t)P(t)$ where $P(t)$ is the solution of *PA*.

If $\beta Q_d(t)P(t) - \tilde{q}_b(t)P(t) < 0$, then we get $Q_d(t)\mu(P(t)) - \tilde{q}_b(t)P(t) < Q_d(t)\mu(0)$. However, *PA* chooses $P(t)$ that maximizes $Q_d(t)\mu(P(t)) - \tilde{q}_b(t)P(t)$ which means $Q_d(t)\mu(P(t)) - \tilde{q}_b(t)P(t) \geq Q_d(t)\mu(0)$ since $0 \in \Pi(t)$. Then we must have $\beta Q_d(t)P(t) - \tilde{q}_b(t)P(t) \geq 0$, i.e., $\beta Q_d(t)P(t) \geq \tilde{q}_b(t)P(t)$. Thus, if $P(t) > 0$, $\tilde{q}_b(t) \leq \beta Q_d(t) \leq \beta(\frac{V}{2} + A_{\text{max}})$; if $P(t) = 0$, by Equation (4), $I_o(t) = 0$, and we also have $M(t) \leq r(t)$, then $\tilde{q}_b(t)$ does not increase anyway. Therefore, $\tilde{q}_b(t) \leq \beta(\frac{V}{2} + A_{\text{max}})$ for all t, which is Equation (10).

Proof of Equation (11): Equation (11) is trivial when $B_b < \infty$. We only need to consider the scenario when $B_b < \infty$. Without loss of generality, let $q_b(0) = 0$. We have the following cases:

i) if $P(t) \geq r(t)$, $I_o(t) = 0$ and $\tilde{q}_b(t) > 0$, then $q_b(t+1) \leq q_b(t)$ and $\tilde{q}_b(t+1) - \tilde{q}_b(t) \leq q_b(t) - q_b(t+1)$, i.e., even if $\tilde{q}_b(t)$ increases, the increment is no larger than the decrement of $q_b(t)$;

ii) if $P(t) < r(t)$, $I_o(t) = 0$ and $\tilde{q}_b(t) > 0$, then $\tilde{q}_b(t) - \tilde{q}_b(t+1) \geq q_b(t+1) - q_b(t)$, i.e., the decrement of $\tilde{q}_b(t)$ is no less than the increment of $q_b(t)$;

iii) if $I_o(t) = 1$, then $q_b(t+1) = r(t) \leq r_{\text{max}}$ by definition of discharging event Equation (4);

iv) if $\tilde{q}_b(t) \leq 0$, by Equation (8), *PA* chooses $P(t) = \min[q_b(t), P_{\text{peak}}]$, then either $q_b(t+1) = r(t) \leq r_{\text{max}}$, or $q_b(t+1) = q_b(t) - P_{\text{peak}} + r(t)$. For the latter case, if $P_{\text{peak}} > r(t)$, then the battery queue state decrease and $\tilde{q}_b(t+1) = q_b(t) - q_b(t+1) = P_{\text{peak}} - r(t) \geq 0$; if $P_{\text{peak}} \leq r(t)$, then $\tilde{q}_b(t) \leq 0$ and case iv) continues. However, $P_{\text{peak}} > \bar{r}$, which means after at most $K < \infty$ slots, $r(t+K) < P_{\text{peak}}$ and the battery queue state starts to decrease.

Combine the above discussion with Equation (10), we have that $\forall t \geq 0$, there $\exists t_1(t), t_2(t) \geq 0$ such that

$$\begin{aligned} q_b(t) &\leq \min[q_b(0), r_{\text{max}}] + |(\tilde{q}_b(t_1(t)))^+ - (\tilde{q}_b(t_2(t)))^+| \\ &\quad + K(r_{\text{max}} - P_{\text{peak}})^+ \\ &\leq (K+1)r_{\text{max}} + \beta(\frac{V}{2} + A_{\text{max}}) < \infty, \end{aligned}$$

which is Equation (11).

Proof of Equation (12): We define the Lyapunov function $L(Q_d(t), \tilde{q}_b(t)) = Q_d^2(t) + \tilde{q}_b^2(t)$, and $\Delta(Q_d(t), \tilde{q}_b(t)) = L(Q_d(t+1), \tilde{q}_b(t+1)) - L(Q_d(t), \tilde{q}_b(t))$.

I) $B_d = \infty$ *and* $B_b = \infty$.

From Equation (7), we have $\tilde{q}_b^2(t+1) \leq (\tilde{q}_b(t) - \eta_o)^2 + (I_o(t) + P(t) - r(t))^2 + 2(\tilde{q}_b(t) - \eta_o)^+(I_o(t) + P(t) - r(t))$. Also from the data queue dynamics, we have $q_d^2(t+1) \leq q_d^2(t) + \mu^2(P(t)) + R^2(t) + 2q_d(t)R(t) - 2q_d(t)\mu(P(t))$, then

$$\begin{aligned} \Delta &= \Delta(q_d(t), \tilde{q}_b(t)) \\ &\leq \mu^2(P(t)) + R^2(t) + 2q_d(t)R(t) - 2q_d(t)\mu(P(t)) + (1 + P_{\text{peak}})^2 + r_{\text{max}}^2 + \eta_o^2 + 2\eta_o r_{\text{max}} + 2\tilde{q}_b(t)(I_o(t) + P(t) - r(t)) \\ &\leq (1 + P_{\text{peak}})^2 + \eta_o^2 + r_{\text{max}}^2 + 2\eta_o r_{\text{max}} + \mu_{\text{max}}^2 + A_{\text{max}}^2(t) + VR(t) + 2\tilde{q}_b(t)(I_o(t) - r(t)) + 2\left[q_d(t) - V/2\right]R(t) - 2\left[q_d(t)\mu(P(t)) - \tilde{q}_b(t)P(t)\right]. \end{aligned}$$

It is apparent that *RC* is trying to minimize the term $\left[q_d(t) - V/2\right]R(t)$, and *PA* is trying to maximize the value of the term $\left[q_d(t)\mu(P(t)) - \tilde{q}_b(t)P(t)\right]$. Since the optimal solution for Problem (A) may not be unique, we let $\mathcal{P}^*$ be the optimal solution set and $P^* \in \mathcal{P}^*$ be any optimal solution, for Problem (A). In time slot t, let $P_m(t)$ be the value that maximize the unconstrained objective function $q_d(t)\mu(P(t)) - \tilde{q}_b(t)P(t)$. Since $\Pi(t) = \{P(t) : 0 \leq P(t) \leq \min[q_b(t), P_{\text{peak}}]\}$, only when $q_b(t) \leq P_{\text{peak}}$ and $P_m(t), P^*(t) \notin \Pi(t)$, we may have $q_d(t)\mu(P(t)) - \tilde{q}_b(t)P(t) \leq q_d(t)\mu(P^*(t)) - \tilde{q}_b(t)P^*(t)$. Thus, we have

$$\begin{aligned} \Delta \leq{}& (1 + P_{\text{peak}})^2 + \eta_o^2 + r_{\text{max}}^2 + 2\eta_o r_{\text{max}} + \mu_{\text{max}}^2 + A_{\text{max}}^2 + V(\\ & R(t) - R^*(t)) + 2q_d(t)[R^*(t) - \mu(P^*(t))] + 2\tilde{q}_b(t)I_o(t) \\ & + 2\tilde{q}_b(t)[P^*(t) - r(t)] + 2[q_d(t)(\mu(P^*(t)) - \mu(P(t))) + \\ & \tilde{q}_b(t)(P(t) - P^*(t))]1_{[P_m(t), P^*(t) \notin \Pi(t)] \bigcap [q_b(t) \leq P_{\text{peak}}]}. \end{aligned}$$

When $P_m(t) \notin \Pi(t)$ and $q_b(t) \leq P_{\text{peak}}$, *PA* will allocate $q_b(t)$ amount of energy for transmission in order to maximize $q_d(t)\mu(P(t)) - \tilde{q}_b(t)P(t)$ within $\Pi(t)$. Under this situation, we must have $I_o(t) = 1$ since $r(t-1) > 0$. Further, $P^*(t) \notin \Pi(t)$ means $P^*(t) \geq P(t)$. Thus,

$$\begin{aligned} \Delta \leq{}& (1 + P_{\text{peak}})^2 + \eta_o^2 + r_{\text{max}}^2 + 2\eta_o r_{\text{max}} + \mu_{\text{max}}^2 + A_{\text{max}}^2 + V(\\ & R(t) - R^*(t)) + 2q_d(t)[R^*(t) - \mu(P^*(t))] + 2\tilde{q}_b(t)[P^*(t) \\ & - r(t)] + (V + 2A_{\text{max}})(\beta + \mu_{\text{max}})I_o(t). \end{aligned} \tag{21}$$

Lemma 1: There exists an optimal policy, under which the actual battery queue is strongly stable, i.e., $\limsup_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} q_b^*(t) < \infty$.

Proof: Suppose there is an optimal policy under which $\limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1} q_b^*(t) = \infty$. This means there exists a subsequence of times $\{t_n\}$ such that $t_n \to \infty$ and $\frac{1}{t_n}\sum_{t=0}^{t_n-1} q_b^*(t) \to \infty$. In other words, $\forall Q > 0,\ \exists t_N \in \{t_n\}$ such that $\forall t_k \in \{t_n\}$ and $t_k \geq t_N$, we have $\frac{1}{t_k}\sum_{t=0}^{t_k-1} q_b^*(t) > Q$.

Let $N(i) = \min\{n : \frac{1}{n}\sum_{t=i}^{i+n-1} r(t) \geq \bar{r}\}$ and $N = \max_{i=0,1,2,\ldots} N(i)$. Note that $N < \infty$, otherwise it contradicts the fact that $\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1} r(t) = \bar{r}$ since $r(t) \leq r_{\max} < \infty,\ \forall t \geq 0$.

Find Q and the associated t_N such that $\frac{1}{t_N}\sum_{t=0}^{t_N-1} q_b^*(t) > Q > N\bar{r}$. Since $t_N < \infty$, $\exists 0 \leq T_0 \leq t_N - 1$ such that $q_b^*(T_0) > Q > N\bar{r}$. Then, if we modify this optimal policy by letting $P^*(t) = \bar{r} \leq P_{\text{peak}},\ \forall t \geq T_0$, and keeping $P^*(t)$ unchanged $\forall t < T_0$, we will not violate the constraint on the battery discharge. Under the modified policy, we have

$$\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\mu(P^*(t)) = \liminf_{T\to\infty}\frac{1}{T}\left[\sum_{t=0}^{T_0-1}\mu(P^*(t)) + \sum_{t=T_0}^{T-1}\mu(\bar{r})\right] = \mu(\bar{r}) \tag{22}$$

Further, by Jensen's inequality, we have

$$\limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\mu(P^*(t)) \leq \mu(\limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}P^*(t)) \leq \mu(\limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}r(t)) = \mu(\bar{r}),$$

which means $\mu(\bar{r})$ is the maximum achievable service rate. Combined with Equation (22), the modified policy can provide the maximum service rate, so with the original optimal sensing rate, the data queue is still strongly stable. Thus, the modified policy is still optimal.

Under the modified optimal policy, $q_b^*(t) \leq r_{\max}T_0 < \infty,\ \forall t < T_0$. Let $M(i) = \min\{n : \frac{1}{n}\sum_{t=i}^{i+n-1} r(t) \leq \bar{r}\}$ and $M = \max_{i=0,1,2,\ldots} M(i) < \infty$. Then $q_b^*(t) \leq r_{\max}T_0 + M(r_{\max} - \bar{r}) < \infty,\ \forall t \geq T_0$. Thus, the battery queue is strongly stable under the modified optimal policy, i.e., $\limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1} q_b^*(t) < \infty$. As a byproduct, $\limsup_{T\to\infty}\frac{q_b^*(T)}{T} = 0$. ■

Note that $q_b^*(t+1) = q_b^*(t) - P^*(t) + r(t)$ for the optimal policy. By multiplying $\tilde{q}_b(t)$ for both sides and rearranging terms, we obtain $\tilde{q}_b(t)[P^*(t) - r(t)] = \tilde{q}_b(t)[q_b^*(t) - q_b^*(t+1)]$. Further, $-r_{\max} \leq \tilde{q}_b(t) \leq \beta(\frac{V}{2} + A_{\max})$. With Lemma 1, by summing from 0 to $T-1$, dividing by T and taking $\liminf_{T\to\infty}$, we have

$$\begin{aligned}
&\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\tilde{q}_b(t)[P^*(t) - r(t)] \\
&= \liminf_{T\to\infty}\frac{\tilde{q}_b(0)q_b^*(0) - \tilde{q}_b(T)q_b^*(T)}{T} \\
&\quad + \liminf_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T} q_b^*(t)(\tilde{q}_b(t) - \tilde{q}_b(t-1)) \\
&\leq (1 + P_{\text{peak}})\liminf_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T} q_b^*(t),
\end{aligned} \tag{23}$$

which is a finite constant that is not related to V.

Note that $q_d^*(t+1) = (q_d^*(t) - \mu(P^*(t)))^+ + R^*(t) \geq q_d^*(t) - \mu(P^*(t)) + R^*(t)$. By multiplying both sides with $q_d(t)$ and rearranging terms, we obtain $q_d(t)(R^*(t) - \mu(P^*(t))) \leq q_d(t)(q_d^*(t+1) - q_d^*(t))$. With the optimal policy, q_d^* is strongly stable. By summing from 0 to $T-1$, dividing by T and taking $\liminf_{T\to\infty}$, we have

$$\begin{aligned}
&\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1} q_d(t)[R^*(t) - \mu(P^*(t))] \\
&\leq \liminf_{T\to\infty}\frac{q_d(T)q_d^*(T) - q_d(0)q_d^*(0)}{T} \\
&\quad + \liminf_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T} q_d^*(t)(q_d(t-1) - q_d(t)) \\
&\leq A_{\max}\liminf_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T} q_d^*(t),
\end{aligned} \tag{24}$$

which is a finite constant that is not related to V.

By summing from 0 to $T-1$, dividing by T and V, taking $\liminf_{T\to\infty}$ over Equation (21), combined with Equation (23), and Equation (24), we get

$$\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1} R(t) \geq \liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1} R^*(t) - \eta_o(\mu_{\max} + \beta) - O(\frac{1}{V}).$$

II) $B_d = \infty$ *and* $B_b < \infty$.
From Equation (6), we have $\tilde{q}_b^2(t+1) \leq (\tilde{q}_b(t) - \eta_o)^2 + (I_o(t) + P(t) - r(t) + M(t))^2 + 2(\tilde{q}_b(t) - \eta_o)^+(I_o(t) + P(t) - r(t) + M(t))$, then

$$\begin{aligned}
\Delta &= \Delta(q_d(t), \tilde{q}_b(t)) \\
&\leq (1 + P_{\text{peak}})^2 + \eta_o^2 + r_{\max}^2 + 2\eta_o r_{\max} + \mu^2(P(t)) + R^2(t) \\
&\quad + VR(t) + 2\left[q_d(t) - V/2\right]R(t) - 2[q_d(t)\mu(P(t)) - \\
&\quad \tilde{q}_b(t)P(t)] + 2\tilde{q}_b(t)(I_o(t) - r(t) + M(t)) \\
&\leq (1 + P_{\text{peak}})^2 + \eta_o^2 + r_{\max}^2 + 2\eta_o r_{\max} + \mu_{\max}^2 + A_{\max}^2 + \\
&\quad V(R(t) - R^*(t)) + 2q_d(t)[R^*(t) - \mu(P^*(t))] + 2\tilde{q}_b(t)[\\
&\quad P^*(t) - r(t) + M^*(t)] + (V + 2A_{\max})(\beta + \mu_{\max})I_o(t) \\
&\quad + 2\tilde{q}_b(t)M(t)
\end{aligned}$$

Without loss of generality, let $q_b(0) = 0$. We have the following cases:

i) if $P(t) \geq r(t)$, $I_o(t) = 0$ and $\tilde{q}_b(t) > 0$, then $M(t) = 0$, $q_b(t+1) \leq q_b(t)$ and $\tilde{q}_b(t+1) - \tilde{q}_b(t) \leq q_b(t) - q_b(t+1)$, i.e., even if $\tilde{q}_b(t)$ increases, the increment is no larger than the decrement of $q_b(t)$;
ii) if $P(t) < r(t)$, $I_o(t) = 0$ and $\tilde{q}_b(t) > 0$, then $\tilde{q}_b(t) - \tilde{q}_b(t+1) = r(t) - P(t) - M(t) - \eta_o + (\eta_o - \tilde{q}_b(t))^+ \geq q_b(t+1) - q_b(t) = r(t) - P(t) - M(t)$, i.e., the decrement of $\tilde{q}_b(t)$ is no less than the increment of $q_b(t)$;
iii) if $I_o(t) = 1$, then $q_b(t+1) = \min[r(t), B_b]$ by definition of discharging event Equation (4). If $M(t) > 0$, then this means $r(t) > B_b$ and $-r(t) + M(t) = -B_b$;
iv) if $\tilde{q}_b(t) \leq 0$, then $\tilde{q}_b(t)M(t) < 0$ anyway.

Combine the above discussion with Equation (10), we have that $\forall t \geq 0$, if $M(t) > 0$ and $\tilde{q}_b(t) > 0$, we must have $\tilde{q}_b(t) \leq \beta(\frac{V}{2} + A_{\max}) + \max[r_{\max}, 1] - B_b$. Thus,

$$\begin{aligned}
&\frac{1}{V} \liminf_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \tilde{q}_b(t)M(t) \\
&\leq \frac{(\beta(\frac{V}{2} + A_{\max}) + \max[r_{\max}, 1] - B_b)^+ r_{\max}}{V} \\
&= O(\frac{(\frac{\beta}{2}V - B_b)^+}{V}).
\end{aligned}$$

The remaining argument is similar to case I), and we obtain

$$\begin{aligned}
\liminf_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} R(t) \geq{}& \liminf_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} R^*(t) - \eta_o(\mu_{\max} + \beta) \\
&- O(\frac{1}{V}) - O(\frac{(\frac{\beta}{2}V - B_b)^+}{V}).
\end{aligned}$$

III) *$B_d < \infty$ and $B_b = \infty$.*
From Equation (7), we have $\tilde{q}_b^2(t+1) \leq (\tilde{q}_b(t) - \eta_o)^2 + (I_o(t) + P(t) - r(t))^2 + 2(\tilde{q}_b(t) - \eta_o)^+(I_o(t) + P(t) - r(t))$. Also from Equation (5), we have $\tilde{q}_d^2(t+1) \leq (\tilde{q}_d(t) - \eta_d R(t))^2 + (I(t) + R(t) - \mu(P(t)))^2 + 2(\tilde{q}_d(t) - \eta_d R(t))^+(I(t) + R(t) - \mu(P(t)))$, then

$$\begin{aligned}
\Delta ={}& \Delta(\tilde{q}_d(t), \tilde{q}_b(t)) \\
\leq{}& (1 + P_{\text{peak}})^2 + \eta_o^2 + r_{\max}^2 + 2\eta_o r_{\max} + \mu^2(P(t)) + (1 + \eta_d^2) \\
&\cdot R^2(t) + VR(t) + 2\tilde{q}_b(t)(I_o(t) - r(t)) + 2\tilde{q}_d(t)I(t) + 2 \\
&[(1 - \eta_d)\tilde{q}_d(t) - V/2]R(t) - 2[\tilde{q}_d(t)\mu(P(t)) - \tilde{q}_b(t)P(t)] \\
\leq{}& (1 + P_{\text{peak}})^2 + \eta_o^2 + r_{\max}^2 + 2\eta_o r_{\max} + \mu_{\max}^2 + (1 + \eta_d^2)A_{\max}^2 \\
&+ VR(t) - VR^*(t) + 2\tilde{q}_d(t)[R^*(t) - \mu(P^*(t)) + I^*(t) - \\
&D^*(t)] + 2\tilde{q}_d(t)[D^*(t) - \eta_d R^*(t)] + 2\tilde{q}_b(t)[P^*(t) - r(t)] \\
&+ 2\tilde{q}_d(t)I(t) + (V + 2A_{\max})(\beta + \mu_{\max})I_o(t)
\end{aligned}$$

Without loss of generality, let $q_d(0) = B_d$. We have the following cases:
i) if $D(t) > 0$, then from $q_d(t+1) = (q_d(t) - \mu(P(t)))^+ + R(t) - D(t)$ and $I(t) = (\mu(P(t)) - q_d(t))^+$, $I(t)$ can be strictly positive only when $A(t) > B_d$. However, whenever $D(t) > 0$, $q_d(t+1) = B_d$.
ii) if $D(t) = 0$, from $q_d(t+1) = q_d(t) - \mu(P(t)) + I(t) + R(t) - D(t)$ and $\tilde{q}_d(t+1) = (\tilde{q}_d(t) - \eta_d R(t))^+ + D(t) - \mu(P(t)) + I(t) + R(t) - D(t)$, $\tilde{q}_d(t)$ decreases no slower and increases no faster than $q_d(t)$.

Note that only when $q_d(t) < \mu_{\max}$, $I(t)$ may be strictly positive. Then combine with the above discussion, we have

$$\begin{aligned}
&\frac{1}{V} \liminf_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \tilde{q}_d(t)I(t) \\
&\leq \frac{(\frac{V}{2} + A_{\max} + \mu_{\max} - B_d)^+ \mu_{\max}}{V} \\
&= O(\frac{(\frac{V}{2} - B_d)^+}{V}).
\end{aligned}$$

Construct an auxiliary queue $\bar{q}^*(t)$ with the following evolution:

$$\bar{q}^*(t+1) = \bar{q}^*(t) - (\eta_d R^*(t) + \epsilon) + D^*(t),$$

where $\epsilon > 0$ can be arbitrarily small. Since for an optimal policy,

$$\begin{aligned}
\limsup_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} D^*(t) &\leq \eta_d \liminf_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} R^*(t) \\
&< \limsup_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} (\eta_d R^*(t) + \epsilon),
\end{aligned}$$

so $\bar{q}^*(t)$ is strongly stable.

Note that $\tilde{q}_d(t)[D^*(t) - \eta_d R^*(t)] = \tilde{q}_d(t)[\bar{q}^*(t+1) - \bar{q}^*(t) + \epsilon]$. By summing from 0 to $T-1$, dividing by T and V, and taking $\liminf_{T\to\infty}$, we have

$$\begin{aligned}
&\frac{1}{V} \liminf_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \tilde{q}_d(t)[D^*(t) - \eta_d R^*(t)] \\
&\leq \frac{1}{V} \liminf_{T\to\infty} \frac{\tilde{q}_d(T)\bar{q}^*(T) - \tilde{q}_d(0)\bar{q}^*(0)}{T} + \frac{\tilde{q}_d(t)}{V}\epsilon \\
&\quad + \frac{1}{V} \liminf_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \bar{q}^*(t)(\tilde{q}_d(t-1) - \tilde{q}_d(t)) \\
&\leq \frac{1}{V} A_{\max} \liminf_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \bar{q}^*(t) + \frac{\epsilon}{V}(\frac{V}{2} + A_{\max}) \\
&= O(\frac{1}{V}) + \frac{\epsilon}{2}.
\end{aligned}$$

The remaining argument is similar to case I). Further, by letting $\epsilon \to 0$, we obtain

$$\begin{aligned}
\liminf_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} R(t) \geq{}& \liminf_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} R^*(t) - \eta_o(\mu_{\max} + \beta) \\
&- O(\frac{1}{V}) - O(\frac{(\frac{V}{2} - B_d)^+}{V}).
\end{aligned}$$

IV) *$B_d < \infty$ and $B_b < \infty$.*
Simply by combing case II) and case III), we obtain

$$\begin{aligned}
\liminf_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} R(t) \geq{}& \liminf_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} R^*(t) - \eta_o(\mu_{\max} + \beta) \\
&- O(\frac{1}{V}) - O(\frac{(\frac{\beta}{2}V - B_b)^+}{V}) - \\
&O(\frac{(\frac{V}{2} - B_d)^+}{V}). \quad \blacksquare
\end{aligned}$$

C. Proof of Corollary 1

Similar to the proof of Proposition 1, we have the fact if $\tilde{q}_b^n(t)$ is strongly stable, then $\limsup_{T\to\infty} \frac{\tilde{q}_b^n(T)}{T} \leq 0$. From Equation (13), we have $\tilde{q}_b^n(t+1) \geq \tilde{q}_b^n(t) - \eta_o^n + \sum_{l\in\Omega_n \bigcup \Theta_n} P_l(t) - r_n(t) + M_n(t) + I_o^n(t)$. Note that $q_b^n(t+1) = q_b^n(t) - \sum_{l\in\Omega_n \bigcup \Theta_n} P_l(t) + r_n(t) - M_n(t)$. By summing from 0 to $T-1$, dividing by T and taking $\limsup$ of both sides, we have

$$\limsup_{T\to\infty} \frac{\tilde{q}_b^n(T)}{T} \geq \lim_{T\to\infty} \frac{\tilde{q}_b^n(0)}{T} - \eta_o^n + \liminf_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} I_o^n(t) + \lim_{T\to\infty} \frac{q_b^n(0) - q_b^n(T)}{T}.$$

Since $\limsup_{T\to\infty} \frac{\tilde{q}_b^n(T)}{T} \leq 0$, $\limsup_{T\to\infty} \frac{\tilde{q}_b^n(0)}{T} = 0$ $\limsup_{T\to\infty} \frac{q_b^n(T)}{T} = 0$, and $\lim_{T\to\infty} \frac{\tilde{q}_b^n(0)}{T} = 0$, so we get $p_o^n = \limsup_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} I_o^n(t) \leq \eta_o^n$, $\forall n \in \mathcal{N}$. ∎

D. Proof of Theorem 2

Proof of Equation (15) and Equation (18): We prove Equation (15) by induction. Let $q_d^{\max}(t)$ be the maximum data queue length for all flows at slot t. Assume that $q_d^{\max}(t) \leq \frac{V}{2} + A_{\max}$ (holds for $t = 0$ by letting $q_d^{n,e}(0) = 0$, $\forall n, e \in \mathcal{N}$), need to show that it holds at slot $t+1$. Consider the data queue $q_d^{n,e}(t+1)$ maintained at any node n for flow destined to any node $e \neq n$ at slot $t+1$. If node n received data destined to e from other node m at slot t, then by the routing policy in Section IV-A and definition of $w_{(m,n)}(t)$, $q_d^{m,e}(t) - q_d^{n,e}(t) > \gamma_{(m,n)}^e$, where (m,n) is the link from node m to node n. Choose γ such that the resulting backlog of the receiving node is no longer than that of the transmitting node (let $\mu_{\max}^{\text{in}}$ to be the maximum endogenous arrivals, then $\gamma = \mu_{\max}^{\text{in}} + A_{\max}$ satisfy this condition), we then have $q_d^{n,e}(t+1) \leq q_d^{n,e}(t) + \gamma$, then $q_d^{n,e}(t+1) \leq q_d^{m,e}(t) \leq q_d^{\max}(t) \leq \frac{V}{2} + A_{\max}$. If node n did not receive any data destined to e from other nodes, then it can only have exogenous arrivals. Clearly $q_d^{n,e}(t+1) \leq q_d^{n,e}(t) \leq \frac{V}{2} + A_{\max}$ if there were no exogenous arrivals. If there were exogenous arrivals, by *AC* of Section IV-A, we must have $q_d^{n,e}(t) \leq \frac{V}{2}$, then $q_d^{n,e}(t+1) \leq \frac{V}{2} + A_{\max}$. Thus, Equation (15) holds. Equation (18) can be shown using the same argument.

Proof of Equation (16) and Equation (19): Since $\mu_l(P_l(t)) \leq \mu_l(0) + \beta P_l(t)$ for $P_l(t) \in \Pi_l(t)$, $\forall t \geq 0$, $w_l(t)\mu_l(P_l(t)) - (\tilde{q}_b^{tran(l)}(t) + \tilde{q}_b^{rec(l)})P_l(t) \leq w_l(t)\mu_l(0) + \beta w_l(t)P_l(t) - (\tilde{q}_b^{tran(l)}(t) + \tilde{q}_b^{rec(l)})P_l(t)$ where $P_l(t)$ is the solution of Equation (14). If $\beta w_l(t)P_l(t) - (\tilde{q}_b^{tran(l)}(t) + \tilde{q}_b^{rec(l)})P_l(t) < 0$, then we get $w_l(t)\mu_l(P_l(t)) - (\tilde{q}_b^{tran(l)}(t) + \tilde{q}_b^{rec(l)})P_l(t) < w_l(t)\mu_l(0)$. However, solution of Equation (14) chooses $P_l(t)$ that maximizes $w_l(t)\mu_l(P_l(t)) - (\tilde{q}_b^{tran(l)}(t) + \tilde{q}_b^{rec(l)})P_l(t)$ which means $w_l(t)\mu_l(P_l(t)) - (\tilde{q}_b^{tran(l)}(t) + \tilde{q}_b^{rec(l)})P_l(t) \geq w_l(t)\mu_l(0)$ since $0 \in \Pi_l(t)$. Then we must have $\beta w_l(t)P_l(t) - (\tilde{q}_b^{tran(l)}(t) + \tilde{q}_b^{rec(l)})P_l(t) \geq 0$. Using a similar argument as in the proof of Equation (10), we obtain $(\tilde{q}_b^{tran(l)}(t) + \tilde{q}_b^{rec(l)}) \leq \beta w_l(t) \leq \beta q_d^{\max} \leq \beta(\frac{V}{2} + A_{\max})$ for all t. This holds for all $l \in \mathcal{L}$, so Equation (16) is proved. Equation (19) can be shown using the same argument.

Proof of Equation (17): Define $L(\vec{q}_d(t), \vec{\tilde{q}}_b(t)) = \sum_{n,e}(q_d^{n,e}(t))^2 + \sum_n(\tilde{q}_b^n(t))^2$, then

$$\begin{aligned}\Delta(t) &= L(\vec{q}_d(t+1), \vec{\tilde{q}}_b(t+1)) - L(\vec{q}_d(t), \vec{\tilde{q}}_b(t)) \\ &\leq \sum_{n,e}[A_{\max}^2 + 2A_{\max}\mu_{\max} + 2\mu_{\max}^2] + \sum_n[(1+P_{\text{peak}})^2 + \\ &\quad (\eta_o^n)^2 + r_{\max}^2 + 2\eta_o r_{\max}] + 2\sum_{n,e} q_d^{n,e}(t)R_n^e(t) - 2\sum_{l\in\mathcal{L}} \\ &\quad \Big[(q_d^{tran(l),e_l(t)}(t) - q_d^{rec(l),e_l(t)}(t))\mu_l(P_l(t)) - (\tilde{q}_b^{tran(l)} \\ &\quad (t) + \tilde{q}_b^{rec(l)}(t))P_l(t)\Big] + 2\sum_n \tilde{q}_b^n(t)\,[I_o^n(t) - r_n(t) + \\ &\quad M_n(t)]\end{aligned}$$

Note that if $q_d^{tran(l),e_l(t)}(t) - q_d^{rec(l),e_l(t)}(t) < \gamma_l^{e_l(t)}$, then $w_l(t) = 0$ and $\mu_l(P_l(t)) = 0$, then $(q_d^{tran(l),e_l(t)}(t) - q_d^{rec(l),e_l(t)}(t))\mu_l(P_l(t)) = w_l(t)\mu_l(P_l(t))$; otherwise, $q_d^{tran(l),e_l(t)}(t) - q_d^{rec(l),e_l(t)}(t) \geq w_l(t)$, then $(q_d^{tran(l),e_l(t)}(t) - q_d^{rec(l),e_l(t)}(t))\mu_l(P_l(t)) \geq w_l(t)\mu_l(P_l(t))$. We then have

$$\begin{aligned}\Delta(t) &\leq \sum_{n,e}[A_{\max}^2 + 2A_{\max}\mu_{\max} + 2\mu_{\max}^2] + \sum_n[(1+P_{\text{peak}})^2 + \\ &\quad (\eta_o^n)^2 + r_{\max}^2 + 2\eta_o r_{\max}] + 2\sum_n \tilde{q}_b^n(t)[I_o^n(t) - r_n(t) + \\ &\quad M_n(t)] + 2\sum_{n,e} q_d^{n,e}(t)R_n^e(t) - 2\sum_{l\in\mathcal{L}}[w_l(t)\mu_l(P_l(t)) - \\ &\quad (\tilde{q}_b^{tran(l)}(t) + \tilde{q}_b^{rec(l)}(t))P_l(t)\Big]\end{aligned}$$

Note that $\sum_{l\in\mathcal{L}}[w_l(t)\mu_l(P_l(t)) - (\tilde{q}_b^{tran(l)}(t) + \tilde{q}_b^{rec(l)}(t))P_l(t)] = \sum_{l\in\mathcal{M}(t)}[w_l(t)\mu_l(P_l^s(t)) - (\tilde{q}_b^{tran(l)}(t) + \tilde{q}_b^{rec(l)}(t))P_l^s(t)] \geq \sum_{l\in\mathcal{M}^*(t)}[w_l(t)\mu_l(P_l^s(t)) - (\tilde{q}_b^{tran(l)}(t) + \tilde{q}_b^{rec(l)}(t))P_l^s(t)]$, where $\mathcal{M}(t)$ is the matching chosen by MWM algorithm and $\mathcal{M}^*(t)$ is the matching picked by optimal policy in slot t, and $P_l^s(t)$, $\forall l \in \mathcal{L}$ are the suppositionally calculated power in *MPA*. Since the objective function of power allocation component is separable over links and for each link it is concave, using similar argument as in single

link case, we have

$$\Delta(t) \le \sum_{n,e}[A_{\max}^2 + 2A_{\max}\mu_{\max} + 2\mu_{\max}^2] + \sum_n[(1+P_{\text{peak}})^2 + (\eta_o^n)^2 + r_{\max}^2 + 2\eta_o r_{\max} + (V+2A_{\max})(\beta+\mu_{\max})I_o^n(t)] + V\sum_{n,e}R_n^e(t) + 2\sum_{n,e}\left[q_d^{n,e}(t) - \frac{V}{2}\right]R_n^{e*}(t) - 2\sum_{n,e}\sum_{l\in\Omega_n}(q_d^{tran(l),e}(t) - q_d^{rec(l),e}(t) - \gamma)\mu_l^e(P_l^*(t)) + 2\sum_n \tilde{q}_b^n(t)\left[\sum_{l\in\Omega_n\bigcup\Theta_n}P_l^*(t) - r_n(t) + M_n^*(t) + M_n(t)\right]$$

$$\le \sum_{n,e}[A_{\max}^2 + 2(A_{\max}+\gamma)\mu_{\max} + 2\mu_{\max}^2] + \sum_n[(1+P_{\text{peak}})^2 + (\eta_o^n)^2 + r_{\max}^2 + 2\eta_o r_{\max} + (V+2A_{\max})(\beta+\mu_{\max}) \cdot I_o^n(t)] + V\sum_{n,e}(R_n^e(t) - R_n^{e*}(t)) + 2\sum_n \tilde{q}_b^n(t)M_n(t) + 2\sum_{n,e}q_d^{n,e}(t)\left[R_n^{e*}(t) + \left(\sum_{l\in\Theta_n} - \sum_{l\in\Omega_n}\right)\mu_l^e(P_l^*(t))\right] + 2\sum_n \tilde{q}_b{}^n(t)\left[\sum_{l\in\Omega_n\bigcup\Theta_n}P_l^*(t) - r_n(t) + M_n^*(t)\right]. \tag{25}$$

Note that $q_b^{n*}(t+1) = q_b^{n*}(t) - \sum_{l\in\Omega_n\bigcup\Theta_n}P_l^*(t) + r_n(t) - M_n^*(t)$ for the optimal policy. By multiplying $\tilde{q}_b^n(t)$ for both sides and rearranging terms, we obtain $\tilde{q}_b^n(t)\left[\sum_{l\in\Omega_n\bigcup\Theta_n}P_l^*(t) - r_n(t) + M_n^*(t)\right] = \tilde{q}_b^n(t)[q_b^{n*}(t) - q_b^{n*}(t+1)]$. By summing from 0 to $T-1$, dividing by T and taking $\liminf_{T\to\infty}$, we have

$$\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\sum_n \tilde{q}_b^n(t)\left[\sum_{l\in\Omega_n\bigcup\Theta_n}P_l^*(t) - r_n(t) + M_n^*(t)\right] = \sum_n \liminf_{T\to\infty}\frac{\tilde{q}_b^n(0)q_b^{n*}(0) - \tilde{q}_b^n(T)q_b^{n*}(T)}{T} + \liminf_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\sum_n q_b^{n*}(t)(\tilde{q}_b^n(t) - \tilde{q}_b^n(t-1)) \le (1+P_{\text{peak}})\liminf_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\sum_n q_b^{n*}(t) \le (1+P_{\text{peak}})\sum_n B_b^n.$$

The argument of case II) in the proof of Theorem 1 can be directly applied here, and we have

$$\frac{1}{V}\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\sum_n \tilde{q}_b^n(t)M_n(t) \le \sum_n O\left(\frac{(\frac{\beta}{2}V - B_b^n)^+}{V}\right).$$

Note that the data queue dynamics can be written as

$$q_d^{n,e}(t+1) = q_d^{n,e}(t) + R_n^e(t) + \sum_{l\in\Theta_n}\mu_l^e(P_l(t)) - \sum_{l\in\Omega_n}\mu_l^e(P_l(t)) - D_{\text{in}}^{n,e}(t) + D_{\text{out}}^{n,e}(t),$$

where $D_{\text{in}}^{n,e}(t)$ is the amount of overestimated endogenous arrivals to the queue $q_d^{n,e}$ since $\sum_{l\in\Theta_n}\mu_l^e(P_l(t))$ may be larger than the actual endogenous arrivals; $D_{\text{out}}^{n,e}(t)$ is the amount of overestimated departures since $\sum_{l\in\Omega_n}\mu_l^e(P_l(t))$ may be larger than the actual departures. Further, at slot t, if node n does not have endogenous arrivals for flow e, then $D_{\text{in}}^{n,e}(t) = 0$; if flow e is transferred from node n to node m, then $D_{\text{out}}^{n,e}(t) = D_{\text{in}}^{m,e}(t)$. By choosing $\gamma = A_{\max} + \mu_{\max}$, the resulting backlog of the receiving node is no longer than that of the transmitting node. We then have

$$\sum_{n,e}q_d^{n,e}(t)(q_d^{n,e*}(t+1) - q_d^{n,e*}(t)) = \sum_{n,e}q_d^{n,e}(t)\left[R_n^{e*}(t) - D_{\text{in}}^{n,e*}(t) + D_{\text{out}}^{n,e*}(t) + \sum_{l\in\Theta_n}\mu_l^e(P_l^*(t)) - \sum_{l\in\Omega_n}\mu_l^e(P_l^*(t))\right] \ge \sum_{n,e}q_d^{n,e}(t)\left[R_n^{e*}(t) + \sum_{l\in\Theta_n}\mu_l^e(P_l^*(t)) - \sum_{l\in\Omega_n}\mu_l^e(P_l^*(t))\right],$$

Since $q_d^{n,e}(t) - q_d^{n,e}(t+1) \le A_{\max} + \mu_{\max}$, $\forall t \ge 0$, $\forall n, e \in \mathcal{N}$ and $q_d^{n,e*}$ are strongly stable $\forall n, e \in \mathcal{N}$, then

$$\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\sum_{n,e}q_d^{n,e}(t)(q_d^{n,e*}(t+1) - q_d^{n,e*}(t)) \le \sum_{n,e}\left[\lim_{T\to\infty}\frac{q_d^{n,e}(T)q_d^{n,e*}(T)}{T} + (A_{\max}+\mu_{\max}) \cdot \liminf_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}q_d^{n,e*}(t) - \lim_{T\to\infty}\frac{q_d^{n,e}(0)q_d^{n,e*}(0)}{T}\right] \le \sum_{n,e}\left[(\frac{V}{2} + A_{\max})\lim_{T\to\infty}\frac{q_d^{n,e*}(T)}{T} + (A_{\max}+\mu_{\max}) \cdot \liminf_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}q_d^{n,e*}(t)\right] \le \sum_{n,e}(A_{\max}+\mu_{\max})\liminf_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}q_d^{n,e*}(t),$$

which is a finite constant that is not related to V. Thus, we have $\frac{1}{V}\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\sum_{n,e}q_d^{n,e}(t)[R_n^{e*}(t) + \sum_{l\in\Theta_n}\mu_l^e(P_l^*(t)) - \sum_{l\in\Omega_n}\mu_l^e(P_l^*(t))] \le O(\frac{1}{V})$. Thus, by summing from 0 to $T-1$, dividing by T and V, taking $\liminf_{T\to\infty}$ for Equation (25), we get

$$\sum_n\left\{\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\sum_e R_n^e(t)\right\} \ge \sum_n\left\{\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\sum_e R_n^{e*}(t) - \eta_o^n(\mu_{\max}+\beta) + O\left(\frac{(\frac{\beta}{2}V - B_b^n)^+}{V}\right)\right\} - O(\frac{1}{V}).$$

which is Equation (17).

Proof of Equation (20): Define $L(\vec{q}_d(t), \vec{\tilde{q}}_b(t)) = \frac{1}{2}\sum_{n,e}(q_d^{n,e}(t))^2 + \frac{1}{2}\sum_n(\tilde{q}_b^n)^2$, and then

$$\begin{aligned}\Delta(t) &= L(\vec{q}_d(t+1), \vec{\tilde{q}}_b(t+1)) - L(\vec{q}_d(t), \vec{\tilde{q}}_b(t)) \\ &\le \sum_{n,e}[\frac{1}{2}A_{\max}^2 + (A_{\max}+\gamma)\mu_{\max} + \mu_{\max}^2] + \frac{1}{2}\sum_n[(1+ \\ &\quad P_{\text{peak}})^2 + (\eta_o^n)^2 + r_{\max}^2 + 2\eta_o r_{\max} + (V+2A_{\max})(\beta+ \\ &\quad \mu_{\max})I_o^n(t)] + V\sum_{n,e}R_n^e(t) + \frac{1}{2}\sum_{n,e}[q_d^{n,e}(t) - V]R_n^{e*}(t) \\ &\quad + \sum_n \tilde{q}_b{}^n(t)\left[\sum_{l\in\Omega_n\bigcup\Theta_n} P_l^*(t) - r_n(t) + M_n^*(t)\right] + \\ &\quad \sum_{n,e} q_d^{n,e}(t)\left[\left(\sum_{l\in\Theta_n} - \sum_{l\in\Omega_n}\right)\mu_l^e(P_l^*(t))I_{[l\in\mathcal{M}(t)]}\right] + \\ &\quad \sum_n \tilde{q}_b^n(t)M_n(t).\end{aligned}$$

Here the difference from the MWM case is that we need to show $\frac{1}{V}\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T}\sum_{n,e}q_d^{n,e}(t)[\frac{1}{2}R_n^{e*}(t) + \sum_{l\in\Theta_n}\mu_l^e(P_l^*(t))I_{[l\in\mathcal{M}(t)]} - \sum_{l\in\Omega_n}\mu_l^e(P_l^*(t))I_{[l\in\mathcal{M}(t)]}] \le O(\frac{1}{V})$. Similar to the MWM case, we only need to show that given that all the data queues are strongly stable under the optimal policy queue evolution, i.e.,

$$\begin{aligned} q_d^{n,e*}(t+1) \le (q_d^{n,e*}(t) - \sum_{l\in\Omega_n}\mu_l^e(P_l^*(t)))^+ + \\ R_n^{e*}(t) + \sum_{l\in\Theta_n}\mu_l^e(P_l^*(t)), \end{aligned} \tag{26}$$

we have all the corresponding data queues strongly stable with the following evolution,

$$\begin{aligned} \bar{q}_d^{n,e*}(t+1) \le (\bar{q}_d^{n,e*}(t) - \sum_{l\in\Omega_n}\mu_l^e(P_l^*(t))I_{[l\in\mathcal{M}(t)]})^+ + \\ \frac{1}{2}R_n^{e*}(t) + \sum_{l\in\Theta_n}\mu_l^e(P_l^*(t))I_{[l\in\mathcal{M}(t)]}. \end{aligned} \tag{27}$$

Let $H^* = \{H_{n,e}^{l*}\}$ denote the routing matrix under the optimal policy, where $H_{n,e}^{l*} = 1$ means the sensing data at node n destined for node e is routed through link l. Let $\bar{\mu}_l^{e*} = \liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\mu_l^e(P_l^*(t))$. Then Equation (26) implies that all the accumulative data queues with the following queue evolutions are all strongly stable:

$$\begin{aligned} Q_d^{n,e*}(t+1) \le (Q_d^{n,e*}(t) - \sum_{l\in\Omega_n}1)^+ + R_n^{e*}(t) \\ + \sum_{l\in\Theta_n}\frac{\sum_{\substack{m\in\mathcal{N}\\ m\neq n}}\sum_e R_m^{e*}(t)H_{m,e}^{l*}}{\bar{\mu}_l^{e*}}. \end{aligned} \tag{28}$$

Note that $I_{[l\in\mathcal{M}(t)]}$ is only related to the state of data queues at slot t, which is a function of $\vec{r}(0), \vec{r}(1), \ldots, \vec{r}(t-1)$ and $\vec{A}(0), \vec{A}(1), \ldots, \vec{A}(t-1)$. $P_l^*(t)$ is the optimal power allocation decision which can be obtained from backward dynamic programming, so it is a function of $\vec{r}(t), \vec{r}(t+1), \ldots$ and $\vec{A}(t), \vec{A}(t+1), \ldots$. Thus, $I_{[l\in\mathcal{M}(t)]}$ and $\mu_l^e(P_l^*(t))$ are independent and then uncorrelated, so

$$\begin{aligned} &\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\mu_l^e(P_l^*(t))I_{[l\in\mathcal{M}(t)]} \\ &= \liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\mu_l^e(P_l^*(t))\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}I_{[l\in\mathcal{M}(t)]} \\ &= \liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\bar{\mu}_l^{e*}I_{[l\in\mathcal{M}(t)]}. \end{aligned}$$

In order to show Equation (27), we only need to show all the accumulative data queues under the MM based algorithm with the following queue evolutions are strongly stable:

$$\begin{aligned} \bar{Q}_d^{n,e*}(t+1) \le (\bar{Q}_d^{n,e*}(t) - \sum_{l\in\Omega_n}I_{[l\in\mathcal{M}(t)]})^+ + R_n^{e*}(t) \\ + \sum_{l\in\Theta_n}\frac{\sum_{\substack{m\in\mathcal{N}\\ m\neq n}}\sum_e R_m^{e*}(t)H_{m,e}^{l*}}{\bar{\mu}_l^{e*}}. \end{aligned} \tag{29}$$

Equation(28) implies Equation (29) by the existing result from [15]. ■